

# Spin structure function $g_1(x, Q^2)$ and the DHGHY integral $I(Q^2)$ at low $Q^2$ : predictions from the GVMD model

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**Abstract:** Theoretical predictions for polarized nucleon structure function  $g_1(x, Q^2)$  at low  $Q^2$  are obtained in the framework of the Generalized Vector Meson Dominance model. Contributions from both light and heavy vector mesons are evaluated. In the photoproduction limit the first moment of  $g_1$  is related to the static properties of nucleon via the Drell-Hearn-Gerasimov-Hosoda-Yamamoto sum rule. This property is employed to fix the magnitude of the light vector meson contribution to  $g_1$ , using the recent measurements in the region of baryonic resonances. Results are compared to the data on  $g_1(x, Q^2)$ . Finally, the DHGHY moment function  $I(Q^2)$  is calculated, and our theoretical predictions are confronted with the recent preliminary data obtained at the Jefferson Laboratory.

## 1 Introduction

Data on polarized nucleon structure function  $g_1(x, Q^2)$  are now available in the region of low values of (negative) four-momentum transfer,  $Q^2$ , [1, 2, 3, 4]. This region is of

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particular interest since nonperturbative mechanisms dominate the particle dynamics there and thus a transition from soft- to hard physics may be studied. In the fixed target experiments the low values of  $Q^2$  are reached simultaneously with the low values of the Bjorken variable,  $x$ , [2, 3], and therefore predictions for spin structure functions in both the low  $x$  and low  $Q^2$  region are needed. Partonic contribution to  $g_1$  which controls the structure function in the deep inelastic domain has thus to be suitably extended to the low  $Q^2$  region. In the  $Q^2=0$  limit,  $g_1$  should be a finite function of  $W^2$ , free from any kinematic singularities or zeroes.

In the previous attempt, [5],  $g_1$  at low  $x$  and low  $Q^2$  was described within a formalism based on the unintegrated spin dependent parton distributions, incorporating the leading order Altarelli–Parisi evolution and the double  $\ln^2(1/x)$  resummation at low  $x$ . The  $\ln^2(1/x)$  effects are not yet significant in the kinematic range of the fixed target experiments but the formalism based on unintegrated parton distributions is very suitable for extrapolating  $g_1$  to the region of low  $Q^2$ . Also a VMD-type nonperturbative part of  $g_1$  was included, its unknown normalisation to be extracted from the data. The model reproduced general trends in the measurements but their statistical accuracy was too low to constrain the VMD contribution. However the nonzero and negative value of the VMD contribution was clearly preferred.

A convenient way of describing structure functions both in the nonperturbative and in the scaling (“asymptotic”) region is to employ the Generalized Vector Meson Dominance (GVMD) model with an infinite number of vector mesons which couple to a virtual photon (cf.[6]). The heavy meson ( $M_V > Q_0$ ) contribution is directly related to the structure function in the scaling region,  $g_1^{AS}$ , described by the QCD improved parton model, suitably extrapolated to the low  $Q^2$  region. The contribution of light ( $M_V < Q_0$ ) vector mesons describes nonperturbative effects and vanishes as  $1/Q^4$  for large  $Q^2$ . At low  $Q^2$  these effects are large and predominant. Here  $M_V$  denotes the mass of a vector meson. The GVMD model was successfully applied to describe the low  $Q^2$  behaviour of the unpolarized structure function  $F_2(x, Q^2)$  [7, 8].

In this paper we apply GVMD to evaluate the nonperturbative contributions to the polarized structure function  $g_1(x, Q^2)$  at low values of  $Q^2$ . We start with the formulation of the GVMD representation of  $g_1(x, Q^2)$  and then specify the representation of the light and of the heavy vector meson components: Vector Meson Dominance (VMD) for the former and asymptotic for the latter one (Section 2). Two different parametrizations are used to describe the VMD part of  $g_1(x, Q^2)$ . The asymptotic part is parametrized using the GRSV fit [9]. Then the Drell-Hearn-Gerasimov-Hosoda-Yamamoto (DHGHY) sum rule [10, 11, 12] together with measurements in the resonance region are employed to

fix the magnitude of the light vector meson contribution to  $g_1$  (Section 3). Numerical results are discussed and compared to other analyses in Section 4. Conclusions and outlook are given in Section 5.

## 2 The GVMD representation of the structure function $g_1(x, Q^2)$ and the DHGHY sum rule

In the GVMD model, the spin dependent nucleon structure function  $g_1$  has the following representation, valid for fixed  $W^2 \gg Q^2$ , i.e. small values of  $x$ ,  $x = Q^2/(Q^2 + W^2 - M^2)$ :

$$g_1(x, Q^2) = g_1^L(x, Q^2) + g_1^H(x, Q^2). \quad (1)$$

The first term:

$$g_1^L(x, Q^2) = \frac{M\nu}{4\pi} \sum_V \frac{M_V^4 \Delta\sigma_V(W^2)}{\gamma_V^2 (Q^2 + M_V^2)^2}, \quad (2)$$

sums up contributions from light vector mesons,  $M_V < Q_0$  where  $Q_0^2 \sim 1 \text{ GeV}^2$  [8]. Here  $W$  is the invariant mass of the electroproduced hadronic system,  $\nu = Q^2/2Mx$ , and  $M$  denotes the nucleon mass. The constants  $\gamma_V^2$  are determined from the leptonic widths of the vector mesons and the cross sections  $\Delta\sigma_V(W^2)$  are combinations of the total cross sections for the scattering of polarised mesons and nucleons. They are not known and have to be parametrized. Following Ref. [5], we assume that they can be expressed through the combinations of nonperturbative parton distributions,  $\Delta p_j^{(0)}(x)$ , evaluated at fixed  $Q_0^2$ :

$$\begin{aligned} \frac{M\nu}{4\pi} \sum_{V=\rho,\omega} \frac{M_V^4 \Delta\sigma_V(W^2)}{\gamma_V^2 (Q^2 + M_V^2)^2} &= C \left[ \frac{4}{9} (\Delta u_{val}^{(0)}(x) + \Delta \bar{u}^{(0)}(x)) + \frac{1}{9} (\Delta d_{val}^{(0)}(x) + \Delta \bar{d}^{(0)}(x)) \right] \frac{M_\rho^4}{(Q^2 + M_\rho^2)^2}, \\ \frac{M\nu}{4\pi} \frac{M_\phi^4 \Delta\sigma_\phi(W^2)}{\gamma_\phi^2 (Q^2 + M_\phi^2)^2} &= C \left[ \frac{1}{9} (2\Delta \bar{s}^{(0)}(x)) \right] \frac{M_\phi^4}{(Q^2 + M_\phi^2)^2}. \end{aligned} \quad (3)$$

It should be noted that the magnitude of the VMD contribution to  $g_1(x, Q^2)$  in (1) is weighted by an unknown constant  $C$ .

The second term in (1),  $g_1^H(x, Q^2)$ , which represents the contribution of heavy ( $M_V > Q_0$ ) vector mesons to  $g_1(x, Q^2)$  can also be treated as an extrapolation of the QCD improved parton model structure function,  $g_1^{AS}(x, Q^2)$ , to arbitrary values of  $Q^2$ . We shall use a simplified representation of  $g_1^H(x, Q^2)$ ,

$$g_1^H(x, Q^2) = g_1^{AS}(\bar{x}, Q^2 + Q_0^2), \quad (4)$$

as it was done in Ref. [8] for the unpolarized structure function  $F_2$ . The scaling variable  $x$  on the right hand side of Eq.(4) is replaced by  $\bar{x} = (Q^2 + Q_0^2)/(Q^2 + Q_0^2 + W^2 - M^2)$ . It follows from equation (4) that  $g_1^H(x, Q^2) \rightarrow g_1^{AS}(x, Q^2)$  as  $Q^2$  is large. Substituting  $g_1^H(x, Q^2)$  in (1) with (4) and  $g_1^L(x, Q^2)$  with a sum of (3) we get

$$\begin{aligned}
g_1(x, Q^2) &= g_1^L(x, Q^2) + g_1^{AS}(\bar{x}, Q^2 + Q_0^2) = \\
&= C \left[ \frac{4}{9}(\Delta u_{val}^{(0)}(x) + \Delta \bar{u}^{(0)}(x)) + \frac{1}{9}(\Delta d_{val}^{(0)}(x) + \Delta \bar{d}^{(0)}(x)) \right] \frac{M_\rho^4}{(Q^2 + M_\rho^2)^2} \\
&+ C \left[ \frac{1}{9}(2\Delta \bar{s}^{(0)}(x)) \right] \frac{M_\phi^4}{(Q^2 + M_\phi^2)^2} \\
&+ g_1^{AS}(\bar{x}, Q^2 + Q_0^2).
\end{aligned} \tag{5}$$

The only free parameter in (5) is the constant  $C$ . Its value may be fixed in the photoproduction limit where the first moment of the structure function  $g_1(x, Q^2)$  is related to static properties of the nucleon via the DHGHY sum rule, cf. [13, 14]. These static properties are the anomalous magnetic moments. For a photon frequency  $\nu$  defined as  $\nu = pq/M$  ( $p$  and  $q$  are energy-momentum four-vectors of the target nucleon and virtual photon respectively, and  $q^2 = -Q^2$ ), the photon-nucleon scattering amplitude,  $S_1(\nu, q^2)$ , fulfills the dispersion relation [13] :

$$S_1(\nu, q^2) = 4 \int_{-q^2/2M}^{\infty} \nu' d\nu' \frac{G_1(\nu', q^2)}{(\nu')^2 - \nu^2}, \tag{6}$$

where  $G_1(\nu, q^2)$  is a polarized nucleon structure function which in the Bjorken limit ( $Q^2, \nu \rightarrow \infty$  at fixed  $x$ ) is related to  $g_1(x, Q^2)$  as :

$$G_1(\nu, q^2) = \frac{M}{\nu} g_1(x, Q^2). \tag{7}$$

As a result of F.E. Low's theorem [15]:

$$S_1(0, 0) = -\kappa_{p(n)}^2, \tag{8}$$

function  $G_1$  fulfills the DHGHY sum rule in the photoproduction limit,  $Q^2 \rightarrow 0$ :

$$\int_0^{\infty} \frac{d\nu}{\nu} G_1(\nu, 0) = -\frac{1}{4} \kappa_{p(n)}^2. \tag{9}$$

Here  $\kappa_{p(n)}$  is the anomalous magnetic moments of proton (or neutron). In the limit  $\nu \rightarrow 0$  the right hand side of equation (6) transforms to :

$$S_1(0, q^2) = 4M \int_{Q^2/2M}^{\infty} \frac{d\nu}{\nu^2} g_1(x(\nu), Q^2). \tag{10}$$

Representation (10) of the scattering amplitude  $S_1(0, q^2)$  is valid down to certain threshold value of  $W_t$ ,  $W_t \gtrsim 2$  GeV. Below  $W_t$  the scattering is dominated by baryonic resonances. We have to separate these two regions in Eq. (10), cf. [13, 14]. The requirement  $W > W_t$  gives the lower limit for integration over  $\nu$  in (10):  $\nu > \nu_t(Q^2)$ , where  $\nu_t(Q^2) = (W_t^2 + Q^2 - M^2)/2M$ . With the DHGHY moment defined as  $I(Q^2) = S_1(0, q^2)/4$ , relation (10) may be rewritten as:

$$I(Q^2) = I_{res}(Q^2) + M \int_{\nu_t(Q^2)}^{\infty} \frac{d\nu}{\nu^2} g_1(x(\nu), Q^2), \quad (11)$$

and the DHGHY sum rule implies that :

$$I(0) = I_{res}(0) + M \int_{\nu_t(0)}^{\infty} \frac{d\nu}{\nu^2} g_1(x(\nu), 0) = -\kappa_{p(n)}^2/4. \quad (12)$$

Here  $I_{res}(Q^2)$  denotes the contribution from resonances. Substituting  $g_1(x(\nu), 0)$  in Eq. (12) by Eq. (5) at  $Q^2 = 0$  and performing the integral in (12), we may obtain the value of the constant  $C$  if  $I_{res}(0)$  is known e.g. from measurements.

### 3 Numerical calculations for the proton

To obtain the value of  $C$  from Eq. (12), the contribution of resonances was evaluated using the preliminary data taken at ELSA/MAMI by the GDH Collaboration [16] at the photoproduction, for  $W_t=1.8$  GeV. Asymptotic part of  $g_1$  was parametrized using GRSV2000 fit for the “standard scenario” of polarized parton distributions with a flavour symmetric light sea,  $\Delta\bar{u} = \Delta\bar{d} = \Delta s = \Delta\bar{s}$ , at the NLO accuracy [9]. The nonperturbative parton distributions,  $\Delta p_j^{(0)}(x)$  in the light vector meson component of  $g_1$ , Eq.(3) were evaluated at fixed  $Q^2 = Q_0^2$ , using, either (i) the GRSV2000 fit, or (ii) a simple, “flat” input:

$$\Delta p_i^{(0)}(x) = N_i(1-x)^{\eta_i} \quad (13)$$

with  $\eta_{u_v} = \eta_{d_v} = 3$ ,  $\eta_{\bar{u}} = \eta_{\bar{s}} = 7$  and  $\eta_g = 5$ . Normalization constants  $N_i$  were determined by imposing the Bjorken sum rule for  $\Delta u_v^{(0)} - \Delta d_v^{(0)}$ , and requiring that the first moments of all other distributions are the same as those determined from the QCD analysis [18]. It was checked that parametrization (13) combined with the unified equations gives reasonable description of the SMC data on  $g_1^{NS}(x, Q^2)$  [19] and on  $g_1^p(x, Q^2)$  [5]. This fit was also used to investigate the magnitude of the double logarithmic corrections,  $\ln^2(1/x)$ , to the spin structure function of proton at low  $x$  [20].

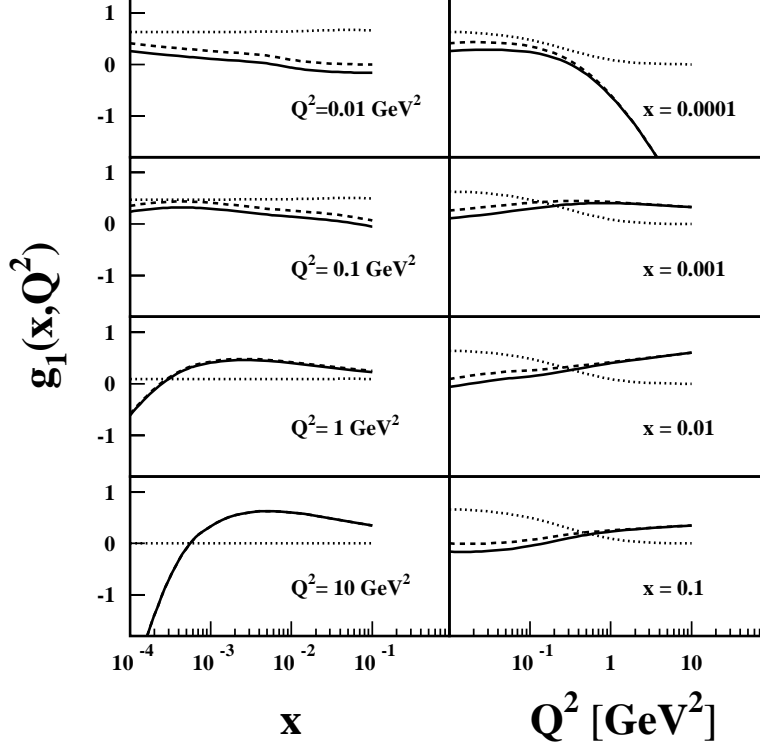


Figure 1: Values of  $g_1$  for the proton as a function of  $x$  and  $Q^2$ . The asymptotic contribution,  $g_1^{AS}$ , is marked with broken lines, the VMD part,  $g_1^L$ , with dotted lines and the continuous curves mark their sum, according to eq. (5).

We have assumed  $Q_0^2 = 1.2 \text{ GeV}^2$ , cf. Eqs. (1) and (3), in accordance with the analysis of  $F_2$ , [7, 8]. As a result the constant  $C$  was found to be  $-0.30$  in case (i) and  $-0.24$  in case (ii). These values change at most by 13% when  $Q_0^2$  changes in the interval  $1.0 < Q_0^2 < 1.6 \text{ GeV}^2$ .

Negative value of the nonperturbative, Vector Meson Dominance, contribution was also obtained within a formalism based on unintegrated spin dependent parton distributions supplemented with the VMD [5] and from the phenomenological analysis of the sum rules [14, 22].

Using the above values of  $C$ , we have calculated  $g_1(x, Q^2)$  from Eq. (5). Results are shown in Fig.1 for different values of  $x$  and  $Q^2$ , separately for the VMD and asymptotic

parts. The VMD decreases fast with  $Q^2$ ; for  $Q^2 \sim 10 \text{ GeV}^2$  the asymptotic function coincides to better than 0.1 % with  $g_1(x, Q^2)$ .

The only measurements performed at low values of  $x$  and  $Q^2$  are those obtained by the SMC with a dedicated low  $x$  trigger, [3]. They are plotted in Fig.2 together with the results of our model, calculated at the  $(x, Q^2)$  values corresponding to those of the SMC. Mean  $Q^2$  at lowest (highest)  $x$  is  $0.02 \text{ GeV}^2$  ( $0.63 \text{ GeV}^2$ ). Kinematic region of that experiment corresponds to  $x < x_t = Q^2/2M\nu_t(Q^2)$ , i.e. it lays outside the baryonic resonance region. Our model reproduces well a general trend in the data; however experimental errors are too large for a more detailed analysis.

We have also computed the DHGHY moment, Eq.(11), for the proton. As a resonance input we used the preliminary results of the JLAB E91-023 experiment [17] for  $0.15 \lesssim Q^2 \lesssim 1.2 \text{ GeV}^2$  and  $W < W_t = W_t(Q^2)$  [21]. Results are shown in Fig.3, separately for contributions from the VMD, from the asymptotic part and from the resonances. Partons contribute significantly even in the photoproduction limit where the main part of the  $I(Q^2)$  comes from resonances.

In Fig. 4 we show our DHGHY moment together with the results of calculations of Refs [22, 23] as well as with the E91-023 measurements in the resonance region used as an input to our  $I(Q^2)$  calculations. We also show the E91-023 data corrected by their authors for the deep inelastic contribution. Our calculations are slightly larger than the DIS-corrected data and then the results of [22] but clearly lower than the results of [23] which overshoot the data.

## 4 Conclusions and outlook

We have analysed the spin dependent structure function  $g_1(x, Q^2)$  at low values of  $Q^2$  and  $x$  in the framework of the Generalised Vector Meson Dominance model. This model was very successful in describing the behaviour of the unpolarized structure function  $F_2$  in the same kinematic region. Contributions from both light- and heavy vector mesons have been evaluated. The latter part of the structure function is directly related to the (QCD improved) parton model  $g_1$  and was evaluated using the GRSV2000 fit in the standard scenario at the NLO accuracy. The light meson contribution represents the nonperturbative part of  $g_1$  and was parametrized either through combinations of the GRSV2000 parton distributions evaluated at fixed  $Q^2 = Q_0^2$ , or through combinations of (almost)  $x$ -independent phenomenological parton distributions. Contribution from the nonperturbative part was fixed using the Drell-Hearn-Gerasimov-Hosoda-

Yamamoto sum rule which is related to the first moment of  $g_1$  in the photoproduction limit. To employ that property an independent information about the resonance region was necessary. We used preliminary measurements of the GDH Collaboration at ELSA/MAMI and of the JLAB E91-023 experiment. As a result the contribution of the nonperturbative part of  $g_1$  was found to be negative and equal to about 0.24 - 0.30, depending on the parametrization. This result changes by at most 13% for the  $Q^2$  in the interval  $1.0 < Q^2 < 1.6 \text{ GeV}^2$ . The final spin dependent structure function  $g_1(x, Q^2)$  reproduces well the trends in the low  $x$ , low  $Q^2$  data of the SMC but for more detailed comparison more precise measurements are needed. Hopefully they will soon be performed by the COMPASS experiment at CERN. We have also computed the DHGHY integral which reproduces well the DIS corrected preliminary measurements by JLAB E91-023.

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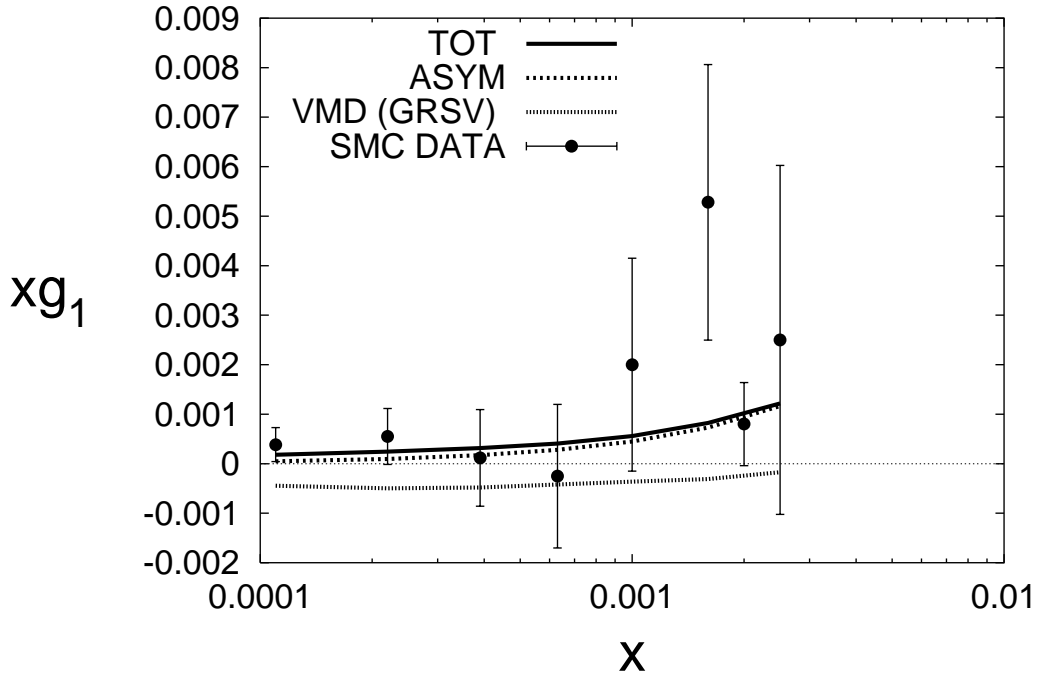
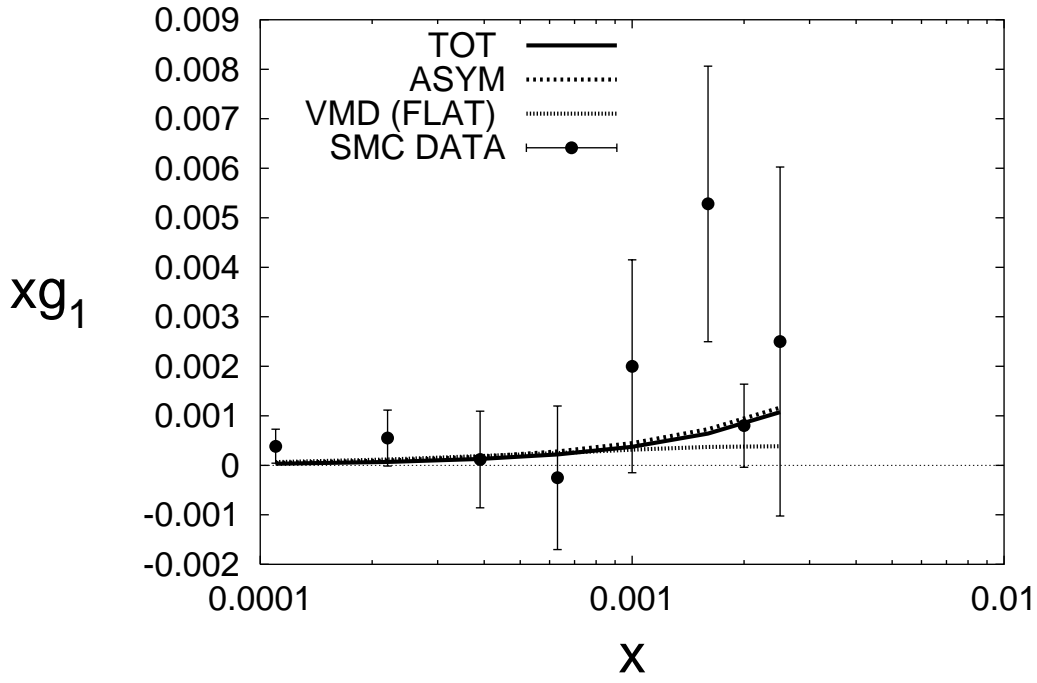


Figure 2: Values of  $xg_1$  for the proton as a function of  $x$  at the measured values of  $Q^2$  in the non-resonant region,  $x < x_t = Q^2/2M\nu_t(Q^2)$ . Upper plot corresponds to the VMD part parametrized using Eq.(13), lower plot corresponds to the GRSV parametrization [9] of the VMD input. The  $g_1^{AS}$  in both plots has been calculated using the GRSV fit for standard scenario at the NLO accuracy. Contributions of the VMD and of the  $xg_1^{AS}$  are shown separately. Points are the SMC measurements at  $Q^2 < 1 \text{ GeV}^2$ , [3]; errors are total. The curves have been calculated at the measured  $x$  and  $Q^2$  values.

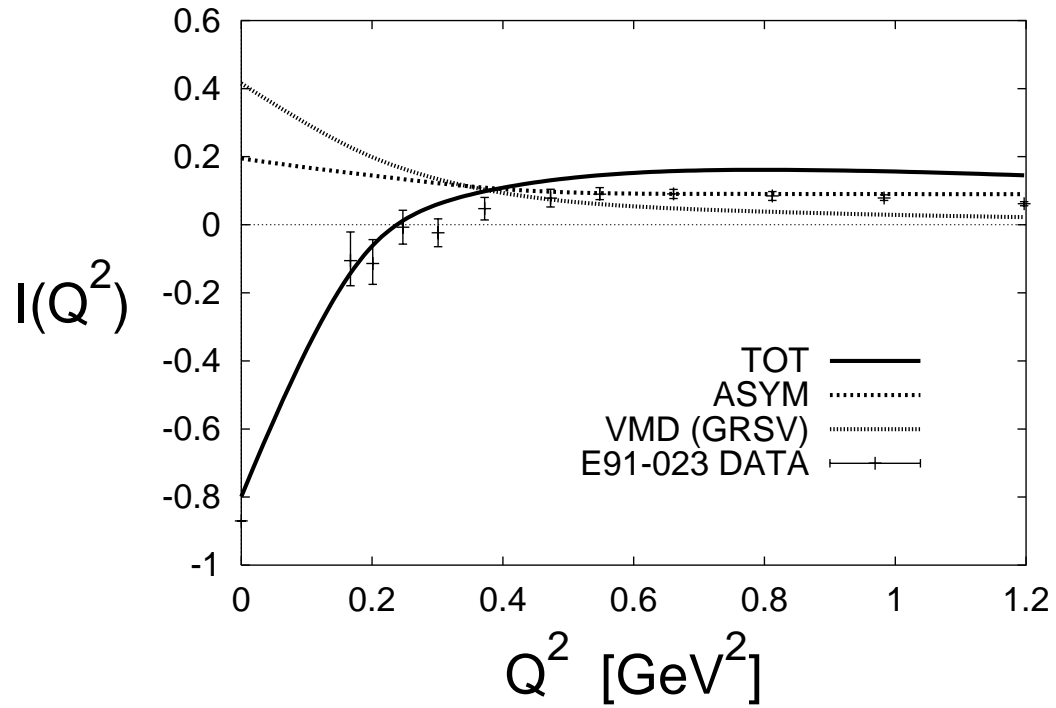
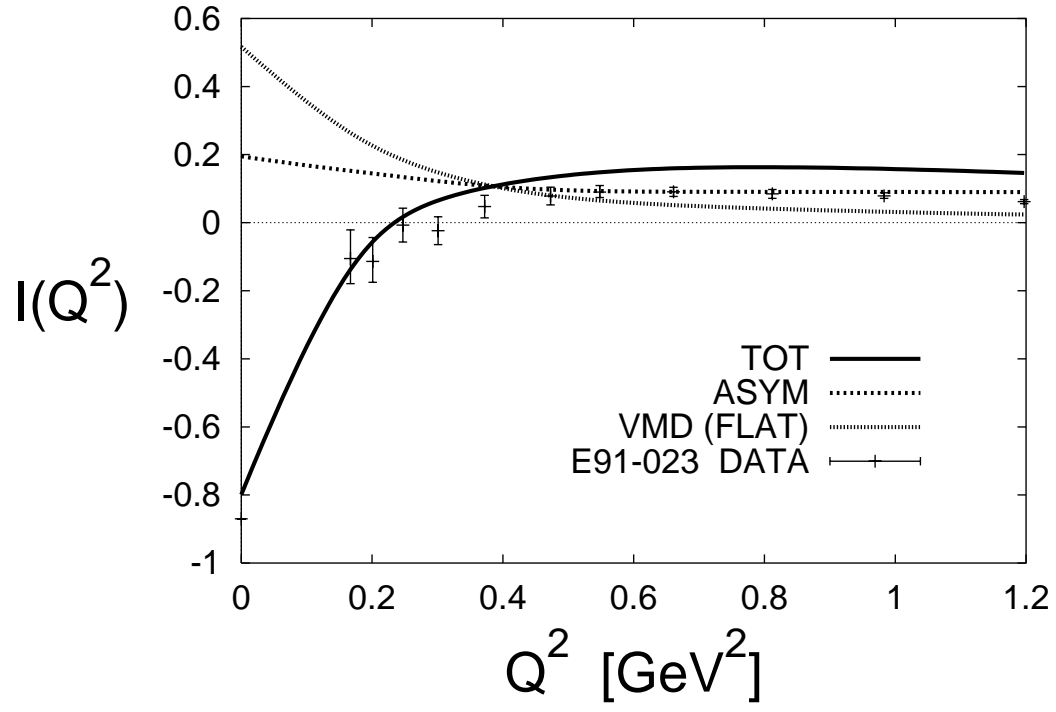


Figure 3: The DHGHY moment  $I(Q^2)$  for the proton. Details concerning the curves are as in Fig.2. Points give the contribution of resonances as measured by the JLAB E91-023 experiment [17] at  $W < W_t(Q^2)$ .

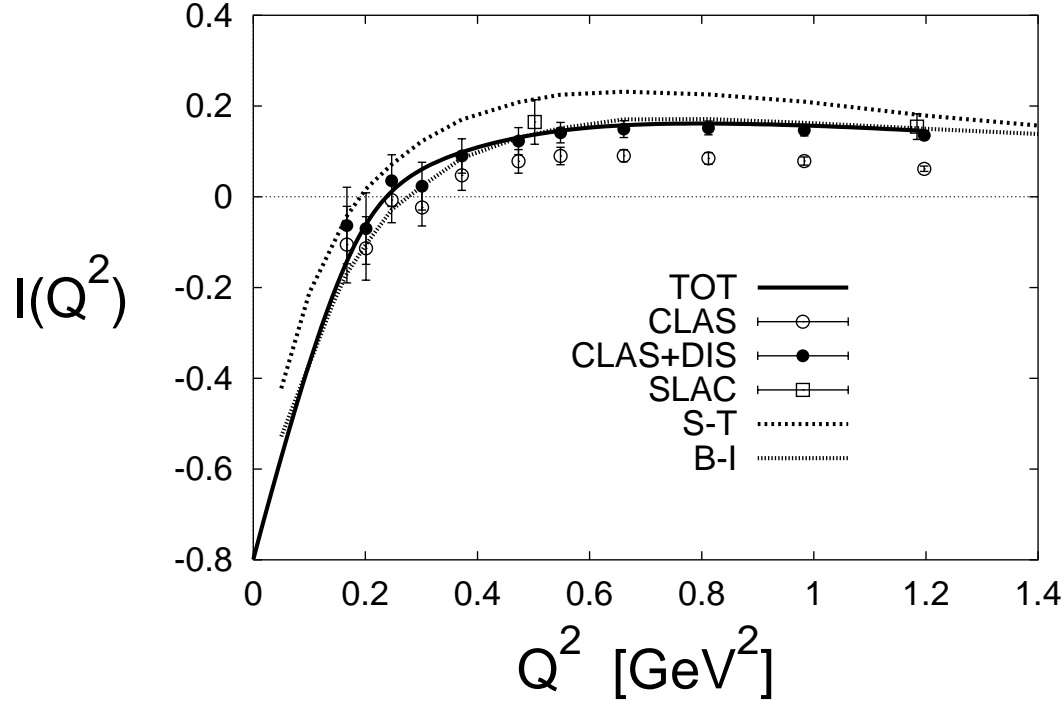


Figure 4: The DHGHY moment  $I(Q^2)$  for the proton with the VMD part parametrized using the GRSV fit [9]. Shown are also calculations of [22] (“B–I”) and [23] (“S–T”). All the points result from the JLAB E91-023 experiment using the Cebaf Large Angle Spectrometer, CLAS [17]. The open circles refer to the resonance region,  $W < W_t(Q^2)$ , the full circles contain a correction for the DIS contribution. Errors are total.